

Arithmetic Encoder/Decoder Example

Assume:

Fixed probabilities : $a=3/12$, $b=4/12$, $c=5/12$

Binary output

8-bit code values

Message sequence = **bac**

Initialization:

low = 0

a: Cum-Freq[3] = 0

First-qtr = 64

b: Cum-Freq[2] = 3

Half = 128

c: Cum-Freq[1] = 7

Third-qtr = 192

Cum-Freq[0] = 12

high = 255

Encode b: range = 256
 $high = 0 + (256 * 7) / 12 - 1 = 148$
 $low = 0 + (256 * 3) / 12 = 64$

Expand Range: $low = 2 * (64 - 64) = 0$
 $high = 2 * (148 - 64) + 1 = 169$

Encode a: range = 170
 $high = 0 + (170 * 3) / 12 - 1 = 41$
 $low = 0 + (170 * 0) / 12 = 0$

Output 01 and scale: $low = 2 * 0 = 0$
 $high = 2 * 41 + 1 = 83$

Output 0 and scale: $low = 2 * 0 = 0$
 $high = 2 * 83 + 1 = 167$

Encode c: range = 168
 $high = 0 + (168 * 12) / 12 - 1 = 167$
 $low = 0 + (168 * 7) / 12 = 98$

Expand Range: $low = 2 * (98 - 64) = 68$
 $high = 2 * (167 - 64) + 1 = 207$

Termination is 10, except we must include the deferred 0 after the 1. Thus the code is 010100. Notice that the probability of the message bac is .03472, and the optimal encoding would require $-\log_2 .03472 = 4.85$ bits. To decode, we begin with code 01010000 = 80 and note that the rightmost 2 bits could be anything.

Decode b: value = 80
range = 256
cum = $((80 - 0 + 1) * 12 - 1 / 256$ = 3
symbol = 2 or b

Encode b: range = 256
high = $0 + (256 * 7) / 12 - 1$ = 148
low = $0 + (256 * 3) / 12$ = 64

Expand value = $2 * (80 - 64) + 0$ = 32
Range: low = $2 * (64 - 64)$ = 0
high = $2 * (148 - 64) + 1$ = 169

Decode a: value = 32
range = 170
cum = $((32 - 0 + 1) * 12 - 1 / 170$ = 2
symbol = 1 or a

Encode a: range = 170
high = $0 + (170 * 3) / 12 - 1$ = 41
low = $0 + (170 * 0) / 12$ = 0

Expand value = $2 * 32 + 0$ = 64
Range: low = $2 * 0$ = 0
high = $2 * 41 + 1$ = 83

Expand value = $2 * 64 + 0$ = 128
Range: low = $2 * 0$ = 0
high = $2 * 83 + 1$ = 167

Decode c: value = 128
range = 168
cum = $((128 - 0 + 1) * 12 - 1 / 168$ = 9
symbol = 3 or c